

# A $[SU(6)]^4$ FLAVOR MODEL WITHOUT MIRROR FERMIONS

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## ABSTRACT

We introduce a three family extension of the Pati-Salam model which is anomaly-free and contains in a single irreducible representation the known quarks and leptons without mirror fermions. Assuming that the breaking of the symmetry admits the implementation of the survival hypothesis, we calculate the mass scales using the renormalization group equation. Finally we show that the proton remains perturbatively stable.

# 1 INTRODUCTION

The renormalizability of the Pati-Salam[1] (PS) model for unification of flavors and forces rest on the existence of conjugate or mirror partners of ordinary fermions. Mirror fermions are fermions with quantum numbers with respect to the Standard Model (SM) gauge group  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  identical to those of the known quarks and leptons, except that they have opposite handedness from ordinary fermions. Their existence vitiate the survival hypothesis[2] according to which chiral fermions that can pair off while respecting a symmetry will do so, acquiring masses grater than or equal to the mass scale of the respected symmetry.

These remarks are illustrated in one of the PS type models. The gauge group for the three-family extension of the PS model is[3]

$$G' \equiv SU(6)_L \otimes SU(6)_R \otimes SU(6)_{CL} \otimes SU(6)_{CR} \times Z_4,$$

where  $\otimes$  indicates a direct product,  $\times$  a semidirect one, and  $Z_4 \equiv (1, P, P^2, P^3)$  is the four-element cyclic group acting upon  $[SU(6)]^4$  such that if  $(A, B, C, D)$  is a representation of  $[SU(6)]^4$  with  $A$  a representation of the first factor,  $B$  of the second,  $C$  of the third, and  $D$  of the fourth, then  $P(A, B, C, D) = (B, C, D, A)$  and then  $Z_4(A, B, C, D) \equiv (A, B, C, D) \oplus (B, C, D, A) \oplus (C, D, A, B) \oplus (D, A, B, C)$ . The charge operator in  $G'$  is defined as[3]

$$Q_{EM} = T_{ZL} + T_{ZR} + Y_{(B-L)L} + Y_{(B-L)R}. \quad (1)$$

The irreducible representation (irrep) of  $G'$  which contains the known fermions is

$$\psi'(144) \equiv Z_4 \psi'(\bar{6}, 1, 6, 1) = \psi'(\bar{6}, 1, 6, 1) \oplus \psi'(1, 6, 1, \bar{6}) \oplus \psi'(6, 1, \bar{6}, 1) \oplus \psi'(1, \bar{6}, 1, 6),$$

where  $\psi'(\bar{6}, 1, 6, 1)$  includes the known left-handed weak doublets while  $\psi'(1, 6, 1, \bar{6})$  includes the known right-handed weak singlets of the three families.  $\psi'(6, 1, \bar{6}, 1)$  and  $\psi'(1, \bar{6}, 1, 6)$  are the mirror fermions of  $\psi'(\bar{6}, 1, 6, 1)$  and  $\psi'(1, 6, 1, \bar{6})$  respectively. With this particle content  $G'$  is free of anomalies because the mirror multiplets cancel the anomalies introduced by the multiplets which contain the known fermions. The model defined with  $G'$  and  $\psi'(144)$  does not have a symmetry that would forbid mass terms of the form  $\psi'(\bar{6}, 1, 6, 1)\psi'(6, 1, \bar{6}, 1) + \psi'(1, 6, 1, \bar{6})\psi'(1, \bar{6}, 1, 6)$  at the  $G'$  scale. The aim of the present work is to introduce a variation of this PS model.

A change in the definition of the permutation operator  $P$  induces a change in  $Z_4$  and therefore in the definition of  $G'$ . The new group,  $G$ , can be written also with the

same  $Z_4$  as in  $G'$  but interchanging the order of the factor groups. In this notation we will consider the gauge group

$$G = SU(6)_L \otimes SU(6)_R \otimes SU(6)_{CR} \otimes SU(6)_{CL} \times Z_4$$

with

$$\psi(144) = Z_4 \psi(\bar{6}, 1, 1, 6) = \psi(\bar{6}, 1, 1, 6) \oplus \psi(1, 1, 6, \bar{6}) \oplus \psi(1, 6, \bar{6}, 1) \oplus \psi(6, \bar{6}, 1, 1).$$

This gauge structure is also free of anomalies but has a different particle content. Indeed, the ordinary fermions in  $\psi(144)$  are included now in  $\psi(\bar{6}, 1, 1, 6) \oplus \psi(1, 6, \bar{6}, 1)$ , but  $\psi(6, \bar{6}, 1, 1) \oplus \psi(1, 1, 6, \bar{6})$  does not contain the mirror fermions of the ordinary fermion fields. To see this let us write the quantum numbers for  $\psi(144)$  with respect to the SM group [our notation designates transformation behavior under  $(SU(3)_C, SU(2)_L, U(1)_Y)$ ]:

$$\begin{aligned} \psi(\bar{6}, 1, 1, 6) &\sim 3(3, 2, 1/3) \oplus 6(1, 2, -1) \oplus 3(1, 2, 1) \\ \psi(1, 6, \bar{6}, 1) &\sim 3(\bar{3}, 1, -4/3) \oplus 3(\bar{3}, 1, 2/3) \oplus 6(1, 1, 2) \oplus 9(1, 1, 0) \oplus 3(1, 1, -2) \\ \psi(6, \bar{6}, 1, 1) &\sim 9(1, 2, 1) \oplus 9(1, 2, -1) \\ \psi(1, 1, 6, \bar{6}) &\sim (8 + 1, 1, 0) \oplus 2(3, 1, 4/3) \oplus 2(\bar{3}, 1, -4/3) \oplus (3, 1, -2/3) \oplus (\bar{3}, 1, 2/3) \oplus \\ &5(1, 1, 0) \oplus 2(1, 1, 2) \oplus 2(1, 1, -2), \end{aligned}$$

where the ordinary left-handed quarks correspond to  $3(3, 2, 1/3)$  in  $\psi(\bar{6}, 1, 1, 6)$ , the ordinary right-handed quarks correspond to  $3(\bar{3}, 1, -4/3) \oplus 3(\bar{3}, 1, 2/3)$  in  $\psi(1, 6, \bar{6}, 1)$ , the known left-handed leptons are in three of the six  $(1, 2, -1)$  of  $\psi(\bar{6}, 1, 1, 6)$ , and the known right-handed charged leptons are in three of the six  $(1, 1, 2)$  of  $\psi(1, 6, \bar{6}, 1)$ . The exotic leptons in  $\psi(\bar{6}, 1, 1, 6)$  belong to the vectorlike representation  $3(1, 2, -1) \oplus 3(1, 2, 1)$  (vectorlike with respect to the SM quantum numbers) and the exotic leptons in  $\psi(1, 6, \bar{6}, 1)$  belong to the vectorlike representation  $3(1, 1, 2) \oplus 3(1, 1, -2) \oplus 9(1, 1, 0)$ , where three lineal combinations of the nine states with quantum numbers  $(1, 1, 0)$  can be identified as the right-handed neutrinos.

Notice that the  $G$  symmetry and the representation content of  $\psi(144)$  forbid mass terms for fermion fields at the unification scale, but according to the survival hypothesis the vectorlike substructures pointed in the former and in the next paragraphs should get masses one scale above  $M_Z$ , the known weak interactions mass scale.

$\psi(6, \bar{6}, 1, 1)$  is formed by 36 exotic Weyl leptons, 9 with positive electric charges, 9 with negative (the charge conjugates to the positive ones), and 18 are neutrals; all together constitute a vectorlike representation. Also all the particles in  $\psi(1, 1, 6, \bar{6})$  form a vectorlike representation, where  $5(1, 1, 0) \oplus 2(1, 1, 2) \oplus 2(1, 1, -2)$  stand for

nine exotic leptons (electric charges  $0, \pm 1$ ),  $2(3, 1, 4/3) \oplus 2(\bar{3}, 1, -4/3)$  refers to two exotic UP type quarks (electric charge  $2/3$ ),  $(3, 1, -2/3) \oplus (\bar{3}, 1, 2/3)$  refers to one exotic DOWN type quark (electric charge  $-1/3$ ), and the nine states in  $(8+1, 1, 0)$  are the most exotic, electrically neutral fermion fields in the model, whose origin and meaning is discussed anon.

The model described by  $[G, \psi(144)]$  (or either by  $[G', \psi'(144)]$ ) unifies the three family SM gauge group, and it unifies also the more general three family chiral color extension of the SM, which has the gauge structure[4]

$$R \equiv SU(3)_{CR} \otimes SU(3)_{CL} \otimes SU(2)_L \otimes U(1)_Y,$$

where the unbroken color group  $SU(3)_C$  of the SM is identified with the diagonal subgroup of  $SU(3)_{CR} \otimes SU(3)_{CL}$ . The model described by  $[G, \psi(144)]$  is an alternative to the PS model for three families and it is a unified theory of a new chiral model with special features, different from the models presented in Refs.[4]. The nine states  $(8+1, 1, 0)$  in  $\psi(1, 1, 6, \bar{6})$  are related to the so-called dichromatic fermion multiplets, belonging to the  $(3, \bar{3})$  representation of the  $SU(3)_{CR} \otimes SU(3)_{CL}$  subgroup of R. Then, according to the nomenclature introduced in Ref.[4],  $(8+1, 1, 0)$  is formed by the “queight”  $(8, 1, 0)$  and the color neutral “quone”  $(1, 1, 0)$ .

Another feature of the model described by  $[G, \psi(144)]$  is that it is the chiral color extension of the vector-color-like model described by  $G^V \equiv SU(6)_L \otimes SU(6)_C \otimes SU(6)_R \times Z_3$  and  $\psi^V(108) = Z_3\psi^V(\bar{6}, 6, 1) = \psi^V(\bar{6}, 6, 1) \oplus \psi^V(6, 1, \bar{6}) \oplus \psi^V(1, \bar{6}, 6)$ . This vector like model was sketched for the first time in Ref.[5] and studied in detail in Refs.[6].  $SU(6)_C$  in  $G^V$  is the diagonal subgroup of  $SU(6)_{CR} \otimes SU(6)_{CL}$  in G, and the particle content of the two models is almost the same in the following sense:  $\psi^V(\bar{6}, 6, 1) = \psi(\bar{6}, 1, 1, 6)$ ,  $\psi^V(1, \bar{6}, 6) = \psi(1, 6, \bar{6}, 1)$ , and  $\psi^V(6, 1, \bar{6}) = \psi(6, \bar{6}, 1, 1)$ . Hence, several techniques used and some results obtained in the study[6] of the structure  $[G^V, \psi^V(108)]$  can be translated to the study of  $[G, \psi(144)]$ .

## 2 SYMMETRY BREAKING

Let us break G down to  $SU(3)_C \otimes U(1)_{EM}$  by the introduction of appropriate elementary Higgs fields which trigger the spontaneous breaking of the symmetry and at the same time produce masses for the fermion fields in  $\psi(144)$ , in such a way that the survival hypothesis[2] holds at each mass scale.

First let us consider the two mass scale symmetry breaking pattern

$$G \xrightarrow{M} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM},$$

with  $M \gg M_Z$ . The running coupling constants of the SM satisfy the one loop Renormalization Group Equations (RGEs)

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i \ln(M/M_Z), \quad (2)$$

where  $\alpha_i = g_i^2/4\pi$ ,  $i = 1, 2, 3$  refers to  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_C$  respectively, and

$$b_i = \left\{ \frac{11}{3} C_i(\text{vectors}) - \frac{2}{3} C_i(\text{Weyl-fermions}) - \frac{1}{6} C_i(\text{scalars}) \right\} / 4\pi, \quad (3)$$

where  $C_i(\dots)$  is the index of the representation to which the (...) particles are assigned (for a complex scalar field the values of  $C_i(\text{scalars})$  should be doubled).

With the normalization of the generators in  $G$  such that  $\alpha_1(M) = \alpha_2(M) = 2\alpha_3(M) = \alpha_{CL}(M) = \alpha_{CR}(M) \equiv \alpha$  (where  $\alpha_{CL(CR)} = g_{CL(CR)}^2/4\pi$  refers to the gauge coupling constant for  $SU(3)_{CL(CR)}$  in R), the relationship

$$\alpha_{EM} = \frac{1}{3} \alpha_2 \sin^2 \theta_W = \frac{3}{19} \alpha_1 \cos^2 \theta_W, \quad (4)$$

where  $\theta_W$  is the weak mixing angle, is valid at all energy scales. This last equation implies also that at all energies

$$3\alpha_{EM}^{-1} = 19\alpha_1^{-1} + \alpha_2^{-1}. \quad (5)$$

From the former equations we get

$$\frac{3}{28} \alpha_{EM}^{-1}(M_Z) = \frac{\alpha_3^{-1}(M_Z)}{2} + \left( \frac{b_3}{2} - \frac{9b_2}{28} - \frac{19b_1}{28} \right) \ln(M/M_Z) \quad (6)$$

and

$$\sin^2 \theta_W(M_Z) = 3\alpha_{EM}(M_Z) \left\{ \frac{\alpha_3^{-1}(M_Z)}{2} + \left( \frac{b_3}{2} - b_2 \right) \ln(M/M_Z) \right\}. \quad (7)$$

After decoupling the vector-like representations in  $\psi(144)$  according to the Appelquist-Carazzone theorem[7] we get:  $2\pi b_3 = [7 - C_3(s)/12]$ ,  $2\pi b_2 = [10/9 - C_2(s)/36]$  and  $2\pi b_1 = -[20/19 + C_1(s)/76]$ , where  $C_i(s)$ ,  $i = 1, 2, 3$  are the indices for the Higgs fields contributing to  $b_i$ . Now, the set of Higgs fields needed to break  $G$  down to  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  and to give at the same time masses to the vectorlike fermions in  $\psi(144)$  contribute negligible to  $C_i(s)$ , because in the effective theory their contribution is highly suppressed by powers of  $M_Z/M$ . So the only Higgs

fields in existence below  $M$  are those which break  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  down to  $SU(3)_C \otimes U(1)_{EM}$ . The simplest set of Higgs fields and Vacuum Expectation Values (VEVs) which do the last breaking and at the same time give rise to mass terms for the known fermion fields is  $\phi_1(72) = \phi_1(6, 1, \bar{6}, 1) \oplus \phi_1(1, \bar{6}, 1, 6) = \phi_\Delta^a \oplus \phi_A^\alpha$  (where  $a, b, \dots; A, B, \dots; \alpha, \beta, \dots; \Delta, \Omega, \dots = 1, \dots, 6$  label  $SU(6)_L$ ,  $SU(6)_R$ ,  $SU(6)_{CL}$  and  $SU(6)_{CR}$  tensor indices respectively), with VEVs  $\langle \phi_1(72) \rangle \neq 0$  in the directions  $(a, \Delta) = (2, 4) = (4, 4) = (6, 4) = (1, 5) = (3, 5) = (5, 5) = (2, 6) = (4, 6) = (6, 6)$ , and  $(\alpha, A) = (4, 2) = (4, 4) = (4, 6) = (5, 1) = (5, 3) = (5, 5) = (6, 2) = (6, 4) = (6, 6)$ . It can be seen that this set is inconsistent with the known quark mass spectrum because it generates see-saw masses for the t and b quarks of the same order of magnitude and proportional to  $M_Z^2/M$ . The alternative is to look for a set of Higgs fields and VEVs which breaks the symmetry and generates at the same time, what is called in Refs.[6] the “*modified horizontal survival hypothesis*”, according to which the t quark gets a mass of order  $M_Z$  via a flavor democratic mass matrix, with the hope that different see-saw mechanisms[8] and radiative corrections[9] reproduce the hierarchy of masses and mixing angles for quarks and leptons. This scenario can be achieved by using, besides  $\phi_1(72)$ , other set of Higgs fields  $\phi_2(1296) = \phi_2(6, \bar{6}, 6, \bar{6}) \equiv \phi_{A, \alpha}^{a, \Delta}$  with VEVs such that  $(a, A) = (2, 2) = (2, 4) = (2, 6) = (4, 2) = (4, 4) = (4, 6) = (6, 2) = (6, 4) = (6, 6)$ , and  $(\Delta, \alpha) = (1, 1) = (2, 2) = (3, 3) = (4, 4) = (5, 5) = (6, 6)$ . ( $\phi_2$  with the VEVs as stated here not only does the job as desired but it also breaks  $SU(3)_{CR} \otimes SU(3)_{CL}$  in R down to  $SU(3)_C$ .)

But how many of the 72 Higgs fields in  $\phi_1$  and of the 1296 Higgs fields in  $\phi_2$  contribute to  $C_i(s)$ ? Let us work with two hypothesis:

*Hypothesis i.* All the Higgs fields in  $\phi_1(72) \oplus \phi_2(1296)$  contribute to  $C_i(s)$ . It is easy to show that in this case the Higgs field contribution to Eqs.(6) and (7) cancels out. (That the contribution of  $\phi_1$  (and also of  $\phi_2$  separately) cancels out in Eqs.(6) and (7) can also be seen from general principles.) Substituting the experimental values[10]  $\sin^2 \theta_W(M_Z) = 0.2341 \pm 0.0025$ ,  $\alpha_{EM}^{-1}(M_Z) = 127.6 \pm 0.2$  and  $\alpha_3(M_Z) = 0.122 \pm 0.005$ , we get from Eq.(6)  $\ln(M/M_Z) = 15.59 \pm 0.31$  while from Eq.(7)  $\ln(M/M_Z) = 15.45 \pm 0.76$ . The compatibility of these results with each other allows us to obtain  $M = 5.5 \times 10^6$  GeVs and to claim that with this hypothesis and with G breaking down to  $SU(3)_C \otimes U(1)_{EM}$  with the SM gauge group as the only intermediate gauge structure, the four coupling constants meet together at a single point M. Unfortunately for this scheme any new physics is at the mass scale  $M \sim 10^6$  GeVs.

*Hypothesis ii.* Only the Higgs fields which develop VEVs contribute to  $C_i(s)$  (hypothesis known in the literature as the “*extended survival hypothesis*”[11]). Under this assumption we get from Eq.(6)  $\ln(M/M_Z) = 12.48 \pm 0.25$  while from Eq.(7)  $\ln(M/M_Z) = 10.89 \pm 0.54$  which are inconsistent solutions.

The other symmetry breaking pattern with only one intermediate mass scale, consistent with present experiments[4] is

$$G \xrightarrow{M} R = SU(3)_{CR} \otimes SU(3)_{CL} \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM},$$

where again  $M >> M_Z$ . To study this case let us write the quantum numbers for  $\psi(144)$  with respect to  $R$ , [now our notation designates transformation behavior under  $(SU(3)_{CR}, SU(3)_{CL}, SU(2)_L, U(1)_Y)$ ]

$$\begin{aligned} \psi(\bar{6}, 1, 1, 6) &\sim 3(1, 3, 2, 1/3) \oplus 6(1, 1, 2, -1) \oplus 3(1, 1, 2, 1) \\ \psi(1, 6, \bar{6}, 1) &\sim 3(\bar{3}, 1, 1, -4/3) \oplus 3(\bar{3}, 1, 1, 2/3) \oplus 6(1, 1, 1, 2) \oplus 3(1, 1, 1, -2) \oplus 9(1, 1, 1, 0) \\ \psi(6, \bar{6}, 1, 1) &\sim 9(2, 1, 1, 1) \oplus 9(2, 1, 1, -1) \\ \psi(1, 1, 6, \bar{6}) &\sim (3, \bar{3}, 1, 0) \oplus 2(1, \bar{3}, 1, -4/3) \oplus (1, \bar{3}, 1, 2/3) \oplus 2(3, 1, 1, 4/3) \oplus (3, 1, 1, -2/3) \oplus \\ &5(1, 1, 1, 0) \oplus 2(1, 1, 1, 2) \oplus 2(1, 1, 1, -2), \end{aligned}$$

where the chiral representations in  $\psi(144)$  with respect to  $R$  are those including the ordinary particles (without right-handed neutrinos) and the new exotic ones with labels  $(3, \bar{3}, 1, 0) \oplus 2(1, \bar{3}, 1, -4/3) \oplus (1, \bar{3}, 1, 2/3) \oplus 2(3, 1, 1, 4/3) \oplus (3, 1, 1, -2/3)$ , all of them belonging to the sector  $\psi(1, 1, 6, \bar{6})$ .

Normalizing the generators in  $G$  as stated before we have that Eqs.(6) and (7) still hold with  $b_3 = b_{3L} + b_{3R}$ , where  $b_{3L}$  and  $b_{3R}$  are related to  $SU(3)_{CL}$  and  $SU(3)_{CR}$  respectively. Then  $C_3(s) = C_{3R}(s) + C_{3L}(s)$ .

$R$  is broken down to  $SU(3)_C \otimes U(1)_{EM}$  by  $\phi_1(72) \oplus \phi_2(1296)$  with the same VEVs as stated before. Now under the hypothesis that all the Higgs fields in  $\phi_1 \oplus \phi_2$  contribute to the beta functions, we have again that the different  $C_i(s)$  contributions cancel out. This time we get from Eq.(6)  $\ln(M/M_Z) = 7.73 \pm 0.15$  while from Eq.(7) we obtain  $\ln(M/M_Z) = 6.27 \pm 0.31$ , which are again incompatible.

On the other hand, the assumption that the extended survival hypothesis[11] holds leads to  $\ln(M/M_Z) = 5.81 \pm 0.11$  from Eq.(6) and to  $\ln(M/M_Z) = 5.78 \pm 0.28$  from Eq.(7) which are consistent solutions. The unification mass scale predicted now is  $M \sim 3.3 \times 10^4$  GeVs. It is evident that this version of the model is rich in experimental consequences.

The following list of comments refers to the model described by  $G$  and  $\psi(144)$  which breaks down to  $SU(3)_C \otimes U(1)_{EM}$  with  $R$  as the only intermediate gauge structure, properly implemented with the survival hypothesis[2], the extended survival

hypothesis[11], and the modified horizontal survival hypothesis[6]:

- The evolution of the four gauge coupling constants in G meet together in a single point at  $M \sim 10^4$  GeVs, in good agreement with precisions data test of the SM.
- The two mass scales  $M \sim 3.3 \times 10^4$  GeVs and  $M_Z \sim 10^2$  GeVs are well within the reach of future experiments.
- The only ordinary fermion which gets a tree level mass of order  $M_Z$  is the t quark. It gets its mass via a flavor democratic mass matrix.
- At the mass scale  $M_Z$  the following exotic particles must exist: 8 “*axigluons*”, two Up type quarks and one Down type quark.
- The *queight* and the *quone* should get masses smaller than  $M_Z$ .
- The gauge fields not related to R and all the other exotic leptons should get masses of order  $M \sim 10^4$  GeVs.

At first glance this version of the model could present the following undesirable features:

- $M \sim 10^4$  GeVs could be a very small unification mass scale (perhaps too close to the present limit for flavor changing neutral currents).
- Since neither  $\phi_1(72)$  or  $\phi_2(1296)$  are able to produce a tree level mass for the *queight* or the *quone*, those particles can pick up only radiative or see-saw masses of a few GeVs (this should be no problem if the *queight* is confined).
- There is not a sufficient large mass scale in the model able to generate see-saw mechanisms[12] for the three neutrinos (most probably  $\nu_e$  remains massless in this scheme as a consequence of the symmetries of the vacuum as in the case of the structure  $[G^V, \psi^V(108)]$  discussed in Ref.[6]).

The above mentioned three problems can be solved by the introduction of new Higgs fields [for example  $\phi_3(1, 1, (\overline{15} + \overline{21}), (15 + 21))$ ] which give tree level masses of order  $M_Z$  to the *queight* and the *quone*. Then we can look for solutions to the RGEs for the symmetry breaking chain

$$G \xrightarrow{M} R \xrightarrow{M_{ch}} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM},$$

with the mass hierarchy  $M > M_{ch} > M_Z$ . With two mass scales to be fixed and a lot of VEVs at our disposal it is possible to look for solutions spanning the range  $10^7$  GeVs  $\geq M > M_{ch} \geq 10^4$  GeVs  $> M_Z \sim 10^2$  GeVs.

Now, independently of the existence of the unifying group G, the set of fermion fields in  $\psi(144)$  which is chiral with respect to R, constitutes an anomaly-free chiral model with only three families, different from the five models (Marks I–V) introduced in Ref. [4]. Such a model deserves a detailed study by its own sake.

### 3 STABILITY OF THE PROTON

In the subspace of the fundamental representation of  $SU(6)_{CR} \otimes SU(6)_{CL}$  the baryon number for G can be associated with the  $12 \times 12$  diagonal matrix

$B = Dg.[(1/3, 1/3, 1/3, 0, 0, 0) \oplus (1/3, 1/3, 1/3, 0, 0, 0)]$ . Since this matrix does not correspond to a generator of G (neither of  $G'$ ), then the baryon number is not gauged in the context of the models discussed here.

Now due to the stated directions of the VEVs for  $\phi_1$  and  $\phi_2$ , it is clear that  $B\langle\phi_1(72)\rangle = B\langle\phi_2(1296)\rangle = 0$ . Therefore B is not broken spontaneously by the set of Higgs fields used for the breaking of R down to  $SU(3)_C \otimes U(1)_{EM}$ . But what about the set of scalars fields used for the breaking of G down to R? It can be shown that it is possible to break  $G \rightarrow R$  using Higgs fields  $\phi_i = Z_4 \phi_i(n_L, n_R, n_{CR}, n_{CL})$  such that  $B\langle\phi_i(n_L, n_R, n_{CR}, n_{CL})\rangle = 0$ , as long as  $n_K = 1, 6, \bar{6}, 15, \bar{15}, 21, \bar{21}, 35$  ( $K = L, R, CR, CL$ ) and as long as the directions for the VEVs of  $SU(6)_{CR} \otimes SU(6)_{CL}$  are such that[13]  $\alpha, \Delta \neq 1, 2, 3$ . Examples of adequate Higgs fields and VEVs are presented for example in Refs.[6]. Our conclusion is that it is possible to choose Higgs fields and VEVs which break  $G \rightarrow SU(3)_C \otimes U(1)_{EM}$ , such that B is not spontaneously broken.

To conclude that B is perturbatively conserved we follow t’Hooft[14] and write B in the space of the fundamental representation for  $SU(6)_{CR} \otimes SU(6)_{CL}$  as  $B = (BL + \Theta)/2$ , where  $BL = Dg.[(1, 1, 1, -1, -1, -1) \otimes (1, 1, 1, -1, -1, -1)]$  is a generator of the G algebra which distinguishes baryon and lepton number, and  $\Theta = Dg.[(1, 1, 1, 1, 1, 1) \otimes (1, 1, 1, 1, 1, 1)]$  generates a  $U(1)_\Theta$  global symmetry of the model. BL and  $\Theta$  are both spontaneously broken, but B is unbroken. A similar situation was analyzed in Ref.[6] for the structure  $[G^V, \psi^V(108)]$ .

## 4 CONCLUSIONS

The model described by  $G'$  and  $\psi'(144)$  was studied originally in Ref.[3] for the symmetry breaking pattern  $G \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$ , under the assumption that all the fermion fields (ordinary and mirrors) contribute to the RGEs and neglecting the contribution of the Higgs fields. Substituting the experimental values[10] of  $\alpha_3(M_Z)$  and  $\alpha_{EM}(M_Z)$  in the results of Refs.[3] we find that the three gauge coupling constants  $\alpha_i, i = 1, 2, 3$  of the SM do not meet at one point; i.e. those results do not satisfy precision tests of the SM. Besides, as we showed in the first section, it is impossible to implement the survival hypothesis in this model due to the fact that  $\psi'(144)$  is vectorlike with respect to  $G'$ .

The new models we have studied here have the same gauge structure as the model in Ref.[3], but a different particle content. As a matter of fact,  $\psi(144)$  does not contain mirror fermion fields and it is not vectorlike with respect to  $G$ . Therefore, the survival hypothesis can be properly implemented at each stage of the symmetry breaking pattern, and the Appelquist-Carazzone[7] theorem can be properly used for the decoupling of heavy fermion fields in the RGEs.

Numerical results were obtained here taking into account not only the decoupling theorem and the survival hypothesis at each stage of the breaking, but also including the effects of the scalar fields. These effects were calculated under two different assumptions and the results were confronted with precision tests of the SM, with the conclusion that under special circumstances the three gauge coupling constants  $\alpha_i, i = 1, 2, 3$  of the SM meet together at the unification scale  $M$  without any intermediate mass scale above  $M_Z$  [i.e. without supersymmetry or extra physics beyond that contained in  $G$  and  $\psi(144)$ ].

The low unification scales discussed here ( $10^7 \text{ GeVs} \geq M \geq 10^5 \text{ GeVs}$ ) do not conflict with data on proton stability because baryon number is perturbatively conserved. Also, lower energy unification makes these models free from problems of grand unified monopoles[15] and the gauge hierarchy problem is also much less severe (no fine tuning required?).

Finally let us see how the known mass spectrum for the elementary fermion fields could be generated in the context of  $[G, \psi(144)]$ :

- The quark  $t$  acquire a tree level mass of order  $M_Z$  by coupling  $\psi(\bar{6}, 1, 1, 6)\psi(1, 6, \bar{6}, 1)$  to  $\phi_2(6, \bar{6}, 6, \bar{6})$  with the VEVs  $\langle\phi_2\rangle$  as stated in Section 2. The  $t$  quark (but not the  $b$  quark) gets its

mass via a flavor democratic mass matrix.

- The b quark and  $\tau$  lepton acquire see-saw masses of order  $M_Z^2/M_{ch}$  by coupling  $[\psi(\bar{6}, 1, 1, 6) + \psi(1, 6, \bar{6}, 1)]\psi(1, 1, 6, \bar{6})$  to  $\phi_1(6, 1, \bar{6}, 1) + \phi(1, \bar{6}, 1, 6)$  with the VEVs  $\langle\phi_1\rangle$  as stated in Section 2.
- Masses for the charged fermion fields in the second and first families can be generated as radiative corrections.

These items are a novel realization of the horizontal survival hypothesis[16] according to which only the heaviest family gets tree level masses from Yukawa couplings. One aspect that the model does not clarify is the observed smallness of the neutrino masses.

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